

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

End Semester Examination – Winter 2018

Course: S.Y.B. Tech (All Branches)

Semester: III

Subject Name: Engineering Mathematics-III

Subject Code: BTBSC301

Max Marks:60

Date:30/11/2018

Duration: 03 Hrs

Instructions to the Students:

1. Attempt Any Five questions of the following. All questions carry equal marks.
2. Use of non-programmable scientific calculators is allowed.
3. Figures to the right indicate full Marks.

Q. 1. a) Show that,

$$\int_0^{\infty} \frac{\sin at}{t} dt = \frac{\pi}{2}. \quad [4]$$

b) Find the Laplace transform of

$$\int_0^t \frac{e^{-3u} \sin 2u}{u} du. \quad [4]$$

c) Find the Laplace transform of the function

$$f(t) = \begin{cases} 2 & , 0 < t < \pi \\ 0 & , \pi < t < 2\pi \\ \sin t & , t > 2\pi \end{cases} \quad [4]$$

Q.2.

a) Find the inverse Laplace transform of $\cot^{-1} \left(\frac{s+3}{2} \right)$. [4]

b) By convolution theorem, find inverse Laplace transform of

$$\frac{s}{(s^2 + 1)(s^2 + 4)}. \quad [4]$$

c) By Laplace transform method, solve the following simultaneous equations

$$\frac{dx}{dt} - y = e^t; \quad \frac{dy}{dt} + x = \sin t; \quad \text{given that } x(0) = 1, y(0) = 0. \quad [4]$$

Q. 3. a) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0 & , |x| > 1. \end{cases} \quad [4]$$

b) Find the Fourier sine transform of $e^{-|x|}$, and hence show that

$$\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx = \frac{\pi e^{-m}}{2}, \quad m > 0. \quad [4]$$

c) Using Parseval's Identity, prove that

$$\int_0^{\infty} \frac{t^2}{(t^2 + 1)^2} dt = \frac{\pi}{4}. \quad [4]$$

Q.4. a) Solve the partial differential equation

$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy. \quad [4]$$

b) Use method of separation of variables to solve the equation

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u; \text{ given that } u(x, 0) = 6e^{-3x}. \quad [4]$$

c) Find the temperature in bar of length 2 units whose ends are kept at zero temperature and lateral surface insulated if initial temperature is

$$\sin\left(\frac{\pi x}{2}\right) + 3 \sin\left(\frac{5\pi x}{2}\right) \quad [4]$$

Q.5. a) If $f(z)$ is analytic function with constant modulus, show that $f(z)$ is constant. [4]

b) If the stream function of an electrostatic field is $\psi = 3xy^2 - x^3$, find the potential function ϕ , where $f(z) = \phi + i\psi$. [4]

c) Prove that the inversion transformation maps a circle in the z -plane into a circle in w -plane or to a straight line if the circle in the z -plane passes through the origin. [4]

Q.6. a) Evaluate $\oint_c \frac{e^z}{z-2} dz$, where c is the circle $|z| = 3$. [4]

b) Evaluate $\oint_c \tan z dz$, where c is the circle $|z| = 2$. [4]

c) Evaluate, using Cauchy's integral formula: [4]

1) $\oint_c \frac{\cos(\pi z)}{(z^2-1)} dz$ around a rectangle with vertices $2 \pm i, -2 \pm i$.

2) $\oint_c \frac{\sin^2 z}{(z-\frac{\pi}{6})^3} dz$, where C is the circle $|z| = 1$.

*** End ***