Dr. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

Summer Supplementary Examinations: May 2018

Branch:	B.Tech (Common to all)	**Semester:**	I
Subject with code:	Engineering Mathematics-I (MATH 101)	**Marks:**	60
Date:	May 02, 2018	**Time:**	03 Hrs.

INSTRUCTION: Attempt any FIVE of the following questions. All questions carry equal marks.

Q.1

(a) Find the rank of the matrix $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ by reducing it to normal form. [6 Marks]

(b) Using Cayley-Hamilton theorem, find A^{-1} where the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. [6 Marks]

Q.2

(a) If $y = x\log(1+x)$, prove that $y_n = \frac{(-1)^{n-2}(n-2)!(x+n)}{(x+1)^n}$. [6 Marks]

(b) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, prove that $x^2y_{n+2} + (2n+1)xy_{n+1} + 2n^2y_n = 0$. [6 Marks]

Q.3 Solve any Two:

(a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$. [6 Marks]

(b) If z is a homogenous function of degree n in x and y, prove that [6 Marks]

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z.$$

(c) If $r^2 = x^2 + y^2 + z^2$ and $V = r^m$, prove that [6 Marks]

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = m(m+1)r^{m-2}.$$

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Q.4

(a) Using Taylor's theorem for two variables, expand the function $f(x,y) = e^x \cos y$ in the powers of $(x-1)$ and $\left(y - \frac{\pi}{4}\right)$. [4Marks]

(b) If the sides and angles of a plane triangle vary in such a way that its circum-radius remains constant, prove that [4Marks]

$$\frac{da}{\cos A} + \frac{db}{\cos B} + \frac{dc}{\cos C} = 0,$$

where da, db and dc are smaller increments in the sides a, b and c, respectively.

(c) If $u = a^3x^2 + b^3y^2 + c^3z^2$ where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, show that the stationary value of u is given by $x = \frac{\sum a}{a}$, $y = \frac{\sum a}{b}$, $z = \frac{\sum a}{c}$. [4Marks]

Q.5 Solve any Two:

(a) Evaluate the integral $I = \int_0^1 \int_{\sqrt{x}}^1 \frac{y^2\,dx\,dy}{\sqrt{y^4 - x^2}}$ by changing the order of integration. [6 Marks]

(b) Evaluate the integral $I = \int_0^4 \int_0^{2\sqrt{z}} \int_0^{\sqrt{4z-x^2}} dy\,dx\,dz$. [6 Marks]

(c) Change to polar co-ordinates to evaluate $I = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$. [6 Marks]

Q.6 (a) State D' Alembert's ratio test, and hence check the convergence of the series: $\sum_{n=1}^{\infty}\left(\frac{n^2}{2^n} + \frac{1}{n^2}\right)$. [6 Marks]

(b) State Cauchy's root test, and hence check the convergence of the series: $\sum \frac{(n+1)^n x^n}{n^{n+1}}$. [6 Marks]

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