

**Instructions to the Students:**

1. Solve ANY FIVE questions out of the following.
2. Use of non-programmable scientific calculators is allowed.
3. Assume suitable data wherever necessary and mention it clearly.

**Q.1 Solve Any Three of the following.**

- A) Let p and q be the propositions “Swimming at the New Jersey shore is allowed” and “Sharks have been spotted near the shore” respectively. Express each of these compound propositions as an English sentence.

- a)  $\neg q$                       b)  $p \rightarrow \neg q$                       c)  $p \leftrightarrow \neg q$                       d)  $\neg p \vee q$

- B) Explain with example, notations used and mathematical expression to describe the following terms.

- i) Membership                      ii) Subset                      iii) Equality of two sets                      iv) Union

- C) Use mathematical induction to show that  $1+5+9+\dots+(4n-3) = n(2n-1)$ ,  $\forall n \geq 1, n \in \mathbb{Z}$

- D) Explain Universal quantifiers and Existential quantifiers with example. What is De Morgan’s law for quantifiers?

**Q.2 Solve the following.**

- A) Check whether the relation R defined in the set  $\{1, 2, 3, 4, 5, 6\}$  is  $R = \{(a, b) : b = a+1\}$  is reflexive, symmetric or transitive. Justify your answer. Find the relation Matrix.

- B) Explain surjective, injective, bijective and inverse function each with example.

**Q.3 Solve Any three of the following.**

- A) Explain the pigeonhole principle with example.

- B) Find how many symbol codes can be formed if the first two symbols are letters and the next three are digits but no symbol is repeated?

- C) What is the expansion of  $(3x + y)^4$ ?

- D) Determine the sequence  $\{a_n\}$  where  $a_n = 3n$  for every non-negative integer, n is a solution of the recurrence relation  $a_n = 2a_{n-1} - a_{n-2}$  for  $n = 2, 3, 4, \dots$

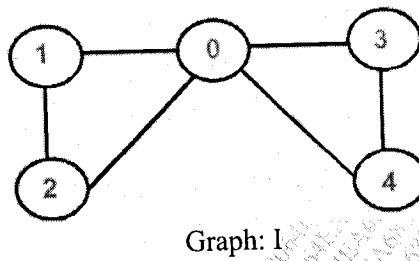
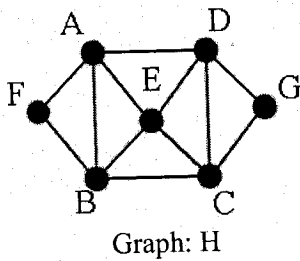
**Q.4 Solve the following.**

- A) Define Euler graph and Hamiltonian Graph.

- i) For a given graph G :

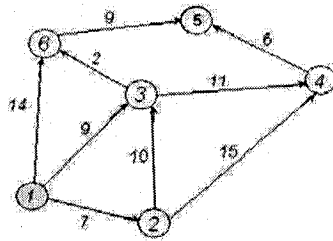
- (a) Find a Hamiltonian path that begins at A and ends at E.
- (b) Find a Hamiltonian circuit that starts at A and ends with the pair of vertices E, A.
- (c) Find a Hamiltonian path that begins at F and ends at G.

- ii) For a given graph I find Eulerian path and Eulerian cycle.



B) Find the shortest path in the given graph using Dijkstra shortest path algorithm.

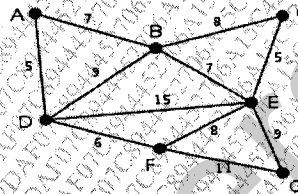
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Q. 5 Solve Any three of the following.

A) Show that a tree with  $n$  vertices has  $n-1$  edges.

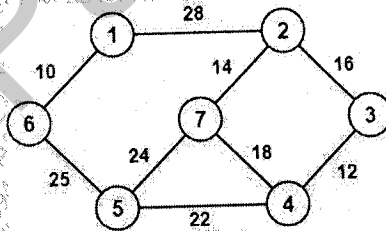
B) Find minimum spanning tree for the given graph using Prim's algorithm?



C) Define the following terms with reference to tree with example.

i) Level and Height of a tree      ii) M-ary Tree      iii) Eccentricity of a vertex

D) Construct the minimum spanning tree (MST) for the given graph using Kruskal's Algorithm.



Q. 6 Solve the following.

A) Define the following terms.

i) Algebraic Structures      ii) Semi Groups      iii) Monoids      iv) Ring  
v) Field      vi) Group

B) For each of the following, determine whether the binary operation  $*$  is commutative or associative?

- $N$  is the set of natural numbers and  $a * b = a + b + 2$  for  $a, b \in N$
- On  $N$  where  $a * b = \min(a, b+2)$
- On  $R$  where  $a * b = a^b$

\*\*\* End \*\*\*