Dr. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

End Semester Examination: May 2018

Branch: B.Tech (Common to all)   **Semester:** II

Subject with code: Engineering Mathematics-II (MATH 201)   **Marks:** 60

Date: 14/05/2018   **Time:** 03 Hrs.

INSTRUCTION: Attempt any FIVE of the following questions. All questions carry equal marks.

Q.1   Solve any three

(a) If $\arg(z+1) = \dfrac{\pi}{6}$ and $\arg(z-1) = \dfrac{2\pi}{3}$, find z.   [4 Marks]

(b) Solve: $x^7 + x^4 + x^3 + 1 = 0$.   [4 Marks]

(c) If $\cos(\theta + i\phi) = \mathrm{Re}^{i\alpha}$, show that $\phi = \dfrac{1}{2}\log_e\left(\dfrac{\sin(\theta - \alpha)}{\sin(\theta + \alpha)}\right)$.   [4 Marks]

(d) Prove that $\tan\left\{i\log\left(\dfrac{a-ib}{a+ib}\right)\right\} = \dfrac{2ab}{a^2 - b^2}$.   [4 Marks]

Q.2   Solve any three.

(a) Solve $(4x - 6y - 1)dx + (3y - 2x - 2)dy = 0$.   [4 Marks]

(b) Solve $\dfrac{dy}{dx} = \dfrac{y+1}{(y+2)e^y - x}$.   [4 Marks]

(c) Solve $(1 + y^2) + (x - e^{\tan^{-1}y})\dfrac{dy}{dx} = 0$.   [4 Marks]

(d) Determine the charge and current at any time 't' in a series R-C circuit with $R = 10\Omega$, $C = 2 \times 10^{-4}$ F and $E = 100V$, given that $q(0) = 0$.   [4 Marks]

Q.3.   Solve any three.

(a) Solve $\dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} + 9y = 5^x - \log 2$.   [4 Marks]

(b) Solve $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 5y = 25x^2$.   [4 Marks]

(c) Solve $(D^2 + 2D + 1)y = e^{-x}\log x$ by method of variation of parameters.   [4 Marks]

(d) Solve $x^2 y'' - 3xy' + 5y = x^2 \sin(\log x)$.   [4 Marks]

Q.4. (a) Obtain the Fourier series expansion of $\sqrt{1-\cos x}$ in the interval $0 \le x \le 2\pi$. [6 Marks]

(b) Find the Half-range co-sine series for $f(x) = \begin{cases} \frac{1}{4} - x & 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \frac{1}{2} < x < 1 \end{cases}$ [6 Marks]

Q.5. (a) If a particle describes the curve $r = 2a\cos\theta$ with constant angular speed ω, find the radial and transverse components of velocity and acceleration. [4 Marks]

(b) For the curve $x = t^3 + 1, y = t^2, z = t$, find the magnitude of tangential and normal components of acceleration at $t = 1$. [4 Marks]

(c) If the particle describes the cardioid $r = a(1 - \cos\theta)$ under a force to the pole, show that the force is proportional to the inverse of the 4th power of the distance. [4 Marks]

Q.6. (a) Find the directional derivative of $\phi = 5x^2 y - 5y^2 z + 2.5z^2 x$ at the point $p(1,1,1)$ in the direction of the line $\dfrac{x-1}{2} = \dfrac{y-3}{-2} = z$. [4 Marks]

(b) If $\vec{F} = (ax + 3y + 4z)\hat{i} + (x - 2y + 3z)\hat{j} + (3x + 2y - z)\hat{k}$ is solenoidal, find the value of 'a'. [4 Marks]

(c) Find the total work done in moving a particle in the force field, given by $\vec{F} = 3xy\,\hat{i} - 5z\,\hat{j} + 10x\,\hat{k}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$. [4 Marks]

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