

**DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY,
LONERE - RAIGAD - 402 103**

SEMESTER EXAMINATION: DECEMBER - 2018

Course: B. Tech (All Branches)

Subject with Subject Code: Engineering Mathematics - I (BTMA101)

Date: 11/12/2018

Marks: 60

Semester: I

Time: 3 Hrs.

Instructions to the Students:-

1. Each question carries 12 marks.
2. All the questions are compulsory.
3. Illustrate your answers with neat sketches, diagram etc., wherever necessary.
4. If some part or parameter is noticed to be missing, you may appropriately assume it and should mention it clearly.

Q.1. Attempt any three

(Marks)

(12)

- (a) Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$ hence find the A^{-1}
- (b) For what values of λ and μ the equations $2x + 3y + 5z = 9, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = \mu$ have (i) no solution (ii) a unique solution or (iii) an infinite number of solutions.
- (c) Use Gauss-Jordan method to find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & -1 \\ 5 & 2 & -3 \end{bmatrix}$
- (d) Find the rank of a matrix A by reducing it to normal form, where $A = \begin{bmatrix} 1 & -12 & 3 \\ 4 & 10 & 2 \\ 0 & 31 & 4 \\ 0 & 10 & 2 \end{bmatrix}$

Q.2. Attempt any three

(12)

- (a) If $z(x+y) = x^2 + y^2$ show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$
- (b) If $u = \sin^{-1} \sqrt{\frac{x^2+y^2}{x+y}}$, Show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4} \tan u [\tan^2 u - 1]$
- (c) If $u = x^2 + y^2, v = 2xy$ & $z = f(u, v)$ then show that $x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 2\sqrt{u^2 - v^2} \frac{\partial z}{\partial u}$
- (d) If $u = x \log xy$ where $x^3 + y^3 + 3xy = 1$, Find $\frac{du}{dx}$

Q.3. Attempt any two

(12)

- (a) $x = \sqrt{vw}, y = \sqrt{uw}, z = \sqrt{uv}$ and $u = r \sin \theta \cos \phi, v = r \sin \theta \sin \phi, w = r \cos \theta$ then find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$
- (b) Expand $f(x, y) = e^x \cos y$ at $(1, \frac{\pi}{4})$ using Taylor's theorem
- (c) Divide 24 into three parts such that the continued product of the first square of the second and cube of the third may be maximum.

Q.4. Attempt any three

(12)

- (a) Evaluate $\int_0^\infty \frac{t^2}{(1+t^2)^2} dt$
- (b) Trace the curve $x = a(\theta + \sin \theta), y = a(1 + \cos \theta)$
- (c) Trace the curve $y^2(2a - x) = x^3$
- (d) Trace the curve $r = 1 + 2 \cos \theta$

Q.5. Attempt any two

(12)

- (a) Change the order of integration and evaluate

$$\int_0^{2a-x} \int_{\frac{x}{a}}^x xy \, dx \, dy$$

- (b) Change to polar co-ordinates and evaluate

$$\int_0^4 \int_{\frac{y^2}{4}}^y \frac{x^2 - y^2}{x^2 + y^2} dx \, dy$$

- (c) Find the area included between the curves $y = x^2 - 6x + 3$ and $y = 2x - 9$ by double integration
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